HEAT TRANSMISSION OF SIMPLE AND COMPOUND WALLS, WITH SPECIAL REFERENCE TO BUILDING CONSTRUCTION

ABSTRACT

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1. Consideration of the theory involved in the transmission of heat through a wall, and the relation between radiation, convection and conduction as involved in the process.

2. The effect of air movement on the film of air in contact with the surface of the wall, and the difference between the air and surface temperatures.

3. Determination of coefficients of transmission, based on inside and outside air temperatures.

1. The transmission of heat through a simple or compound wall, such as may be used in practice for the exterior walls of buildings, is a phenomenon of very general and practical interest. The calculation of the amount of heat transmitted in this way becomes one of the determining factors in proportioning any sort of heating or refrigerating system, and also serves as a ready means of comparing the relative heat insulating efficiencies of any form of standard wall construction.

2. The theoretical data on radiation, convection and conduction available in this field has not been found readily applicable to conditions as they actually exist in practice, as the following considerations will show. In the first place we are

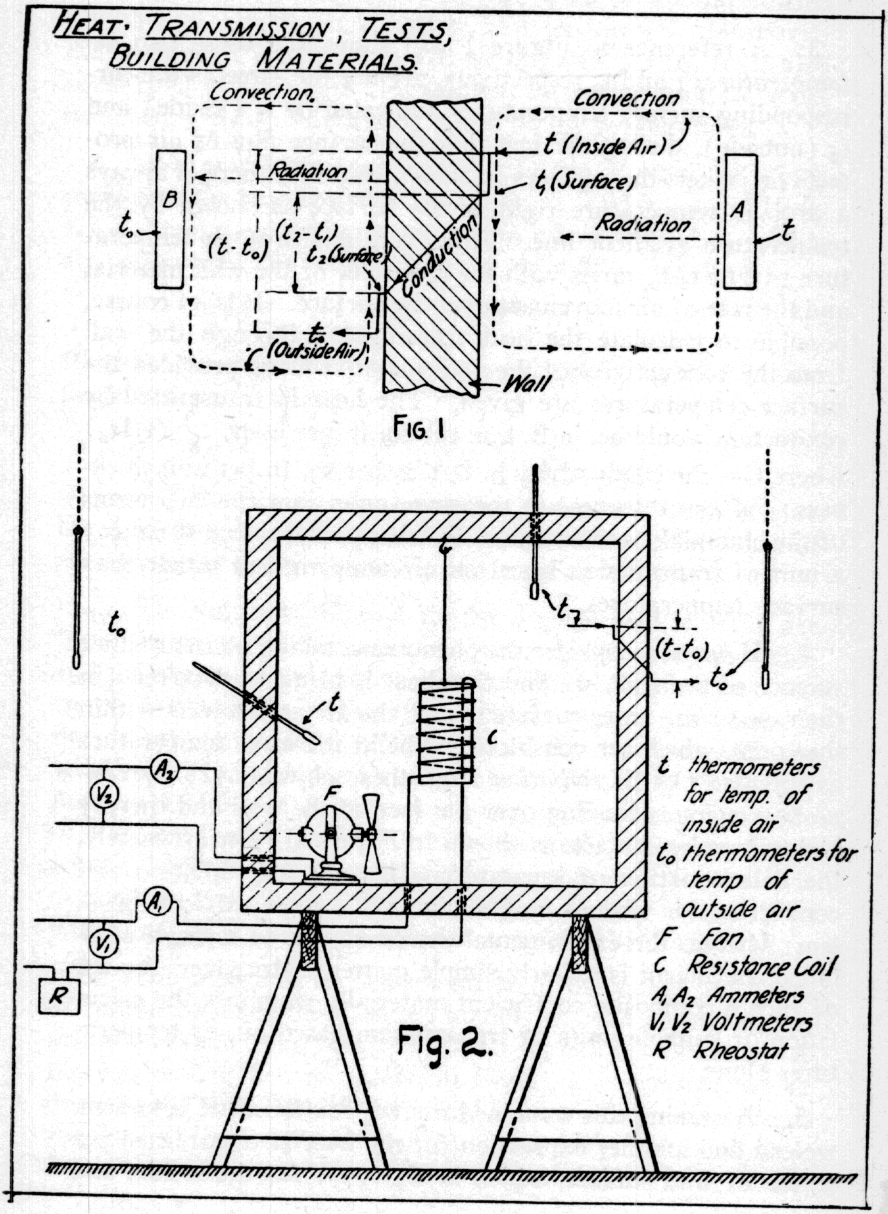

FIG. 1

Fig. 2.

forced to base our calculation on the *air temperatures* on the two sides of the wall since in any problem of this sort we are only concerned with maintaining some fixed or desired internal "room" temperature when a given "outside" temperature exists.

3. A reference to Figure 1 will show that these two *air temperatures* t and t_0 respectively, are not the same as the corresponding surface temperatures, indicated by t_1 (inside) and t_2 (outside), due to the fact that the surface film of air protects or jackets the surfaces in such a way that there is always a drop in temperature right at the surface as shown by the temperature gradient line. Moreover, this drop in temperature $t-t_1$, or t_2-t_0 varies with the character of the wall material and the rate of air movement over the surface. It is, of course, possible to calculate the heat transmission through the wall from the conductivity of the wall material alone provided the surface temperatures are given. The heat H transmitted by conduction would be, in B. t. u. per sq. ft. per hour, $_{x}$ $(t_1 1 t_2)$, where C= the conductivity in B. t. u. per sq. ft. per unit thickness, and x= thickness in the same units; but the two terms of the binomial are unknown. In other words we must employ a unit of transmission based on air temperatures rather than surface temperatures.

4. If we now consider the phenomena taking place at either surface of the wall, we find that heat is being transferred (in the case of the inner surface) from the air and objects within the room, which are considered to be at the same temperature as the air; (1) by *radiation* from these objects; (2) by *convection* currents moving over the face of the wall and thereby losing heat by contact, as shown in Figure 1. Unfortunately, the determination of separate coefficients for radiation and convection for various wall surfaces is a most difficult problem; whereas the experimental determination of a single combined coefficient is a fairly simple matter. Moreover, the use of such a composite coefficient materially simplifies the calculation of suitable units of transmission based on air temperatures alone.

5. Assuming this combined surface coefficient K is known, we can find another expression for the heat H transmitted per sq. ft. of wall surface, which is K_1 $(t-t_1)$ for the inside, and

K_2 (t_2-t_0) for the outside surface, since all the heat entering the wall by *radiation* and *convection* per sq. ft. of inside surface must pass through it by *conduction* and then be discharged from the outside surface by *radiation* and *convection*. It is assumed, of course, that the wall has come to a condition of equilibrium, and is transmitting heat uniformly. Moreover, the amount of heat transmitted by the wall per sq. ft. is also equal to U (t-t_0) where U is the unit of transmission to be determined, already referred to, based on air temperatures.

6. We now have four expressions for H, each of which represents the heat transmission per sq. ft., and involving U, C. K_1, K_2, and the four temperatures, in which only U, t_1, and t_2 are unknown. By elimination of t_1 and t_2 we find

$$U = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{X}{C}}$$

for a simple wall, and for a compound wall in an exactly similar manner we obtain

$$U = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{X_1}{C_1} + \frac{X_2}{C_2} + \frac{X_3}{C_3} + \text{etc.}}$$

where K_1 and K_2 are the respective inside and outside combined surface coefficients, and X is the thickness in inches of each material and C the corresponding conductivity.

7. The values of K_1 and K_2 for a given wall material are found to vary with the rate of air movement over the wall, and the results of tests show that K_1 (inside or still-air coefficient) is practically constant so long as the air movement is due to convection only. The value of K_2 increases with the wind velocity, and for brick ranges from 2.38 K_1 at 5 miles per hour to 4.22 K_1 at 20 miles per hour, with an average value of 3 K_1 at 13 miles per hour, which represents our mean winter wind velocity. It is therefore apparent that once this ratio is determined for any building material, it is only necessary to find values of C and K_1 in order to compute U for any wall.

8. The experimental determination of the values of U can, of course, be made on a limited number of wall constructions, and at the same time values of C and K_1 can be obtained. A thermal testing box as shown in Figure 2 is constructed of the material to be investigated, and a heating element of high resistance wire is centrally located within same. A small desk fan is used to maintain a uniform temperature all over the in-

EXAMPLES IN THE CALCULATION OF HEAT TRANSMISSION OF VARIOUS CONSTRUCTIONS

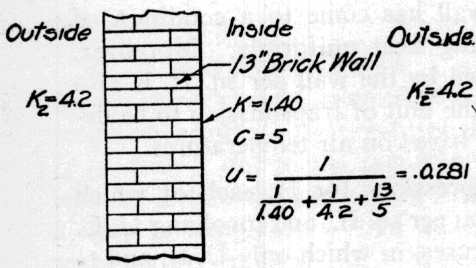

Fig. 3

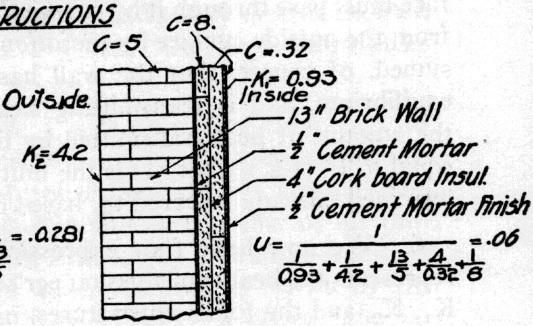

Fig. 4

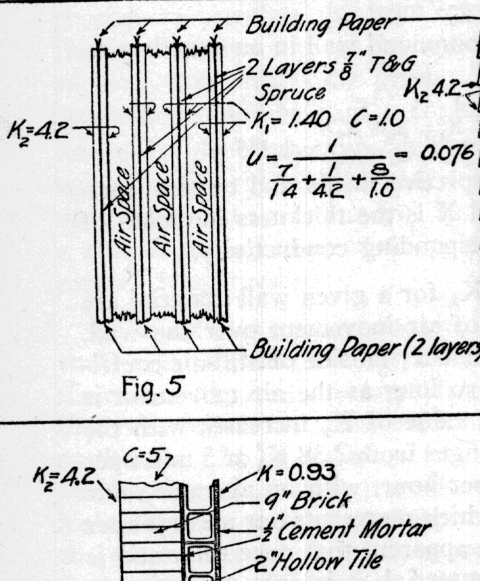

Fig. 5

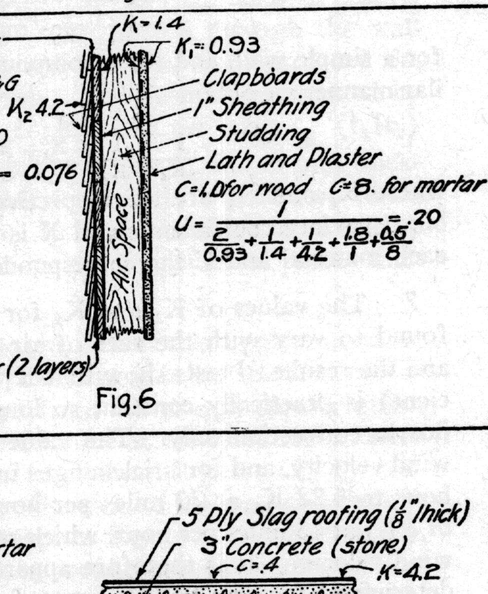

Fig. 6

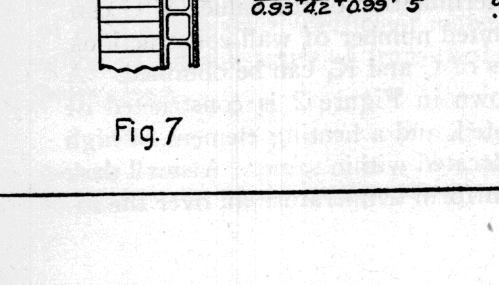

Fig. 7

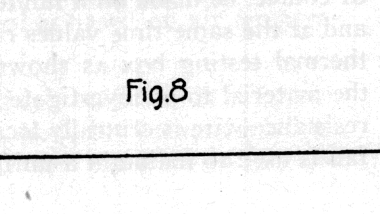

Fig. 8

terior of the box by suitable circulation of the air in the box. Direct current is supplied both coil and fan, and the total input in watts is determined from the ammeter and voltmeter readings. After the box has been under heat for at least 24 hours, and a condition of equilibrium has been established, it is only necessary to make one set of readings and by substitution in the equation US $(t\text{-}t_0)=3.415\ (W_1+W_2)$ find the value of U. The right hand member is, of course, the heat equivalent of the watts supplied, where W_1 and W_2=watts per hour supplied to fan and coil respectively. S=mean area of the six sides of the box.

9. Values of C and K_1 are determined at the same time by the use of thermo-couples, imbedded just in the surface of the wall materials, and used for measuring the surface temperatures t_1 and t_2. Thus, for the determination of conductivity, we have

$$(t_1\text{-}t_2)=3.415\ (W_1+W_2)$$

where the value of C is to be found per inch of thickness, and X+thickness inches. In a similar manner we may find values of K (still air) by using the outside surface temperature of the box, which is standing in still air, or

$$K_1S(t_2\text{-}t_0)=3.415\ (W_1+W_2).$$

10. Since it is manifestly impossible to test all forms of wall construction, it will, in general, be necessary to determine the value of U (the coefficient based on inside and outside air temperatures) by calculation. The equations already derived provide the means of doing this if values for the conductivity C and surface coefficients K_1 and K_2 are known. Tests now in progress in the Mechanical Engineering Laboratory of the University of Illinois, have, as one of their objects, determination of such data, and the figures numbered from 3 to 8 show application of this data to typical simple and compound wall constructions in solving for proper values of U for use in practice. Heat transmission tests on actual walls, such as here shown, give results, which agree very closely with the calculated values.

11. In practical application it is only necessary to multiply the coefficient U by the temperature range and then by the net area of the wall through which the heat loss takes place. Thus, for 70° inside air, and 0° outside air temperature, the total transmission loss for 1000 square feet of wall, such as shown in Fig. 3 is 0.291x(70—0)x1000=20370 B. t. U. per hour.

FATIGUE OF METALS UNDER REPEATED STRESS

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The failure of metal under repeated stress is a familiar phenomenon. Illustrations of such failure are furnished by the bending of a wire back and forth until it breaks; by the failure which takes place in railroad rails after a large number of trains have passed over them; and by the failure of boiler plates between riveted holes after the boiler has heated and cooled many times. The failure of metal under repeated stress is called failure by "fatigue." The old theory of such failures was that under repeated stress metal "crystalized" and became brittle, finally snapping between crystals. This belief lead many engineers to consider wrought iron to be superior to steel under repeated stresses, because wrought iron seemed fibrous in its structure while steel was crystaline.

The use of the microscope in studying metals has very generally discredited the "crystallization" theory. Under the microscope, the structure of all metals is seen to be crystaline, and no marked change in size of crystals can be detected in metal which has failed under repeated stress. The appearance of the fracture of metal to the naked eye is not a reliable indication of the structure of these metals. After a piece of soft steel is broken by a gradually applied tension the fracture will appear silky, not crystalline. If a piece of the same soft steel is nicked and bent it will break in two at the nick and the fracture will appear crystalline. If a piece of the same soft steel bar is bent back and forth a great many times it will finally snap in two with very little warning and the fracture will appear crystalline. The appearance of the fracture is dependent not only on the nature of the metal but upon the shape of the piece broken and the manner of applying load.

Examination under the microscope gives some idea of what happens when metal fails by fatigue. Figure 1*a* shows the appearance under the microscope of an unstressed piece of Norway iron. It is made up of crystals of pure iron and fibres of slag. Figure 1*b* shows the appearance of the same piece of iron after it has been subjected to several hundred repetitions of stress. Right across crystals appear fine lines; these are known as "slip lines" and indicate the splitting up of the crystal. Figure 1*c* shows the appearance of the same piece of iron after several hundred more repetitions of stress. The slip lines are more numerous than in Figure 1*b* and more crystals are "infected" by slip lines. Figure 1*d* shows the appearance of the same piece of metal just before it broke under repeated stress. The slip lines had become very numerous and at *aa* had spread until a crack had formed between the crystals. Shortly after the formation of this crack the piece failed.

The effect of this progressive failure of metal is to weaken the section of a bar, just as a nick cut into it would do, and explains why under repeated stress failure takes place suddenly just as it does when a nicked bar of metal is bent.

The problem which faces the engineer is to design members so that they will not fail under repeated stress. In general the smaller the stresses the less the danger of failure by repeated stress. A common idea concerning metals is that there exists an absolute elastic limit below which metal is absolutely elastic and below which no amount of repeated stress can injure material. It can not be stated positively whether such an absolute elastic amount exists, but in tests under repeated stress failure has occurred at stresses less than the elastic limit has commonly determined by refined testing methods.[1]

The best method of determining safe stresses for metals under repeated stress seems, to the writer, to be the direct experimental study of test specimens subjected to known stresses of varying magnitude repeated many times until failure occurs. Fig. 2 is plotted from the results of such a series of tests, and its general form is typical. Two methods of interpreting the results of such a curve are in use; in one it is assumed that the curve becomes horizontal, and from the test data a horizontal asmyptote to the curve is drawn by estimation, and the stress-ordinate of this asymptote taken as the endurance or "fatigue" limit for the metal. In the other method the assumption of a horizontal asymptote is discarded, and an attempt made to find some simple form of equation which fits the test data. For a wide range of test results of fatigue

[1]The method of determining the elastic limit of a material is to apply known loads to a specimen of the material and then to release the loads. Measurements of length of specimen are made before and after the application and release of each load. When any change in length can be detected after release of load, the elastic limit has been reached.

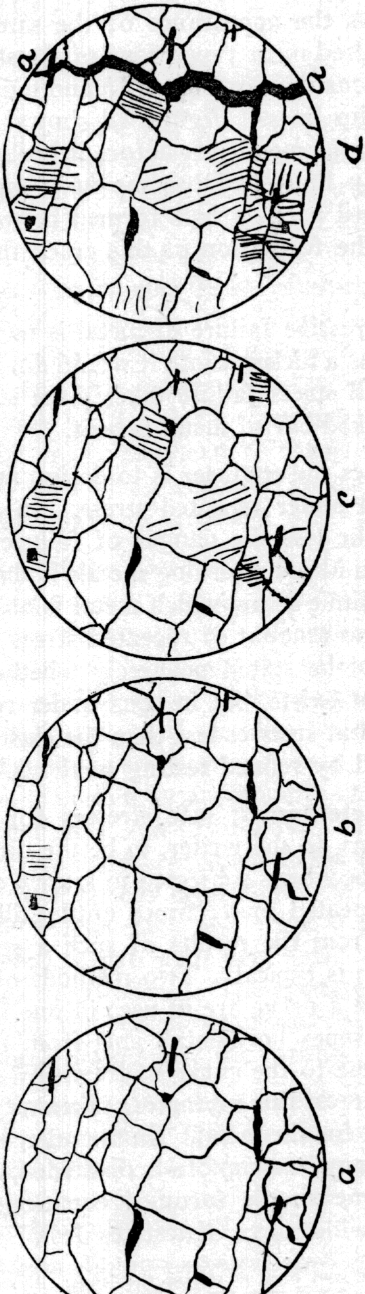

FIG. 1

From photomicrographs made in the Materials Testing Laboratory of the University of Illinois by H. R. Thomas tests the general form of equation,

$$S=AN^{-m} \quad \text{.................(1)}$$

seems to fit test results fairly well,[2] giving stresses for large numbers of repetitions which are somewhat lower than test results, and hence being on the safe side. In the above equation S is the fiber stress in pounds per square inch, N, the number of repetitions of stress necessary to cause failure, and A and m experimentally determined constants.

If the repeated stress on a metal is completely reversed there is much more danger of fatigue failure than if the stress varies from zero to a maximum in one direction. An examination by Mr. F. B. Seely and the writer, of the available published data on repeated stress tests, led to the proposed modification of equation (1) by the separation of the factor A into two parts: one denoted by B, an experimentally determined constant for a material, and the other denoted by $\frac{1}{1-Q}$, dependent on the range of stress to which the material is subjected, Q being the ratio of the minimum stress to the maximum. For completely reversed stresses Q is equal to —1, for stress varying from zero to a maximum, Q is equal to zero.

From the examination of available test data, including data for tests by various experimenters, tests with various kinds of testing machines, and tests of various sizes and shapes of test piece Mr. F. B. Seely and the writer have proposed for metals under repeated stress the general formula[3]

$$S=\frac{B}{(1-Q)\sqrt[8]{N}} \quad \text{....(2)}$$

For very high values of N this formula seems to give stresses somewhat lower than shown by test results; however, the test data for high values of N are so meager that, as the formula is on the safe side, no modification is recommended for general use.

A more convenient form of equation (2) for general use is

$$\log S=\log B-\log(1-Q)-0.125 \log N \quad \text{..................................(3)}$$

The accompanying table gives values of the constant B determined from a study of test data. In using the table, equation (2), or equation (3) a word of caution is necessary. In no case should the stress be taken higher than the safe stress

[2]So far as the writer is aware this form of equation for repeated stresses was first proposed by Professor Basguin of Northwestern University in 1910.

[3]See proceedings of American Society for Testing Materials for 1915 and for 1916, Moore and Seely on Repeated Stress. Also "Text-book of Materials of Engineering," by H. P. Moore, p. 169.

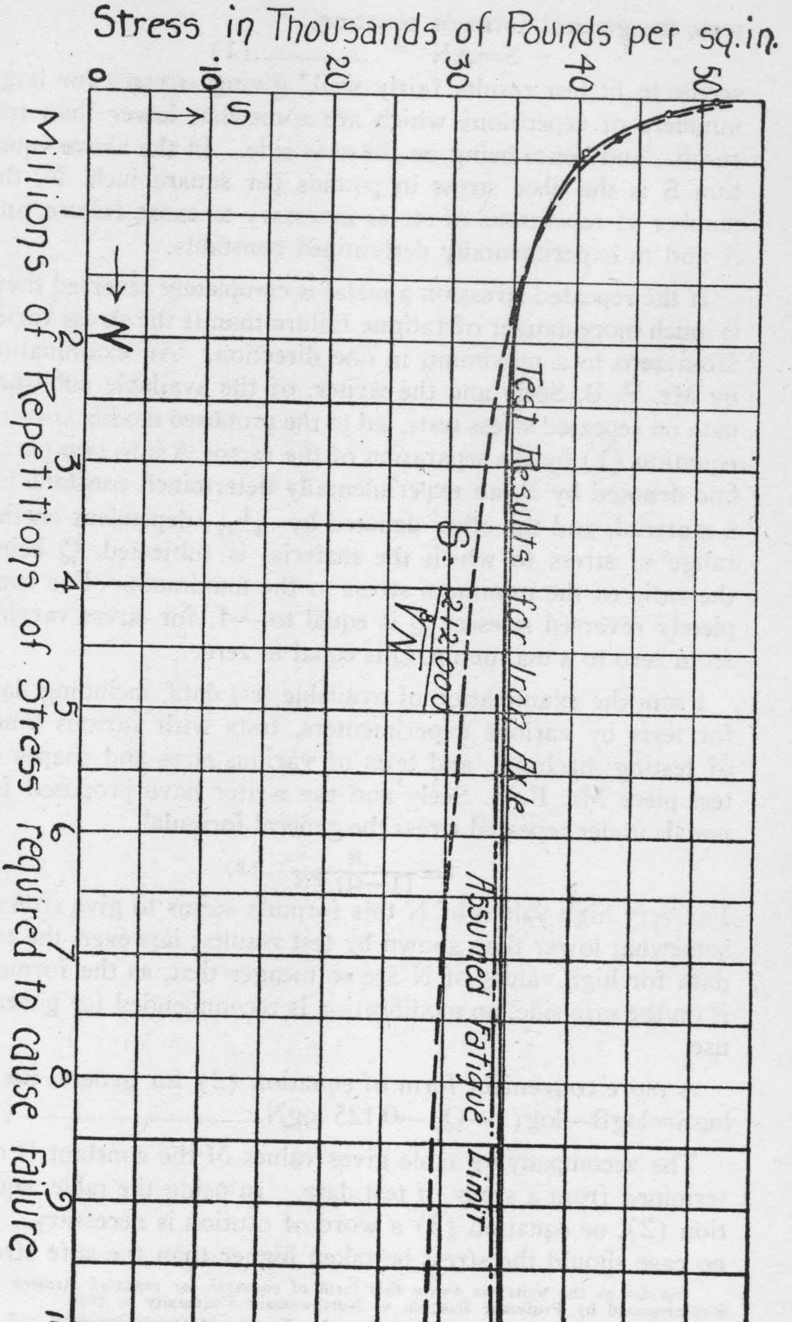
Stress in Thousands of Pounds per sq. in.
0
10
20
30
40
50
S
N
2
3
4
5
6
7
8
9
10
Test Results for Iron Axle
$S = \frac{212000}{\sqrt[8]{N}}$
Assumed Fatigue Limit
Millions of Repetitions of Stress required to cause failure

FIG. 2

under static load as given by engineers' hand books or as determined from test results. If the use of equation (2) or equation (3) with a proper factor of safety gives higher results than the safe static stress, it means that static stress conditions are the determining conditions, and that there is more danger of failure by static stress than by fatigue.

TABLE OF VALUES OF THE CONSTANT B.

Material	B
Structural Steel and Soft Machinery Steel	250,000
Wrought Iron	250,000
Steel, 0.45 per cent Carbon	350,000
Cold-rolled steel Shafting	400,000
Tempered Spring Steel	400,000 to 800,000