THE DETERMINATION OF "g"

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It is not the object of this paper to present a new method or even to consider exhaustively the older methods of determining "g". That would require more time than I have at my disposal. What I hope to do is to indicate a way of getting better results from one of the well known methods.

Of the various methods of determining "g", the simple pendulum method is doubtless the most widely used, and perhaps, all things considered, the most generally satisfactory.

I think it is the method which succeeds best in the hands of our students, and I also think it has within it the possibilities of great precision.

The quantities to be measured are two in number, the length of the pendulum, l, and the period, or half period, t, in the formula

$$g = \frac{\pi^2 l}{t^2}$$

The length of a pendulum about a meter long can easily be measured to within one part in ten thousand, with a good cathetometer, if the pendulum is properly constructed, and if suitable corrections are made for the mass of the suspension and for the moment of inertia of the ball. If, then, the period can be determined with equal accuracy, we should have no difficulty in getting the fourth figure in the final value of "g".

The period, however, must be *squared* and must therefore be determined to within one part in twenty thousand, in order that the value of "g" may be correct to within one part in ten thousand.

To count twenty thousand oscillations and guarantee the count would be rather too great a strain on human endurance, even if we could get a pendulum to continue swinging long enough, which would be another difficulty.

The coincidence method furnishes a perfectly splendid way of supplementing human endurance at this point, but our next difficulty lies in the fact that the oscillations are not strictly isochronous. Many of our laboratory manuals say they are practically so, if the amplitude of the pendulum is not more than five degrees. Well, that might do for measuring t to within one part in a thousand, but not for one part in twenty thousand.

Figure 1 shows the interval between coincidences on the Y axis, plotted against the number of intervals between coincidences on the X axis. It is obvious that as the number of intervals increases, that is, as the amplitude of the pendulum decreases, the length of the interval decreases.

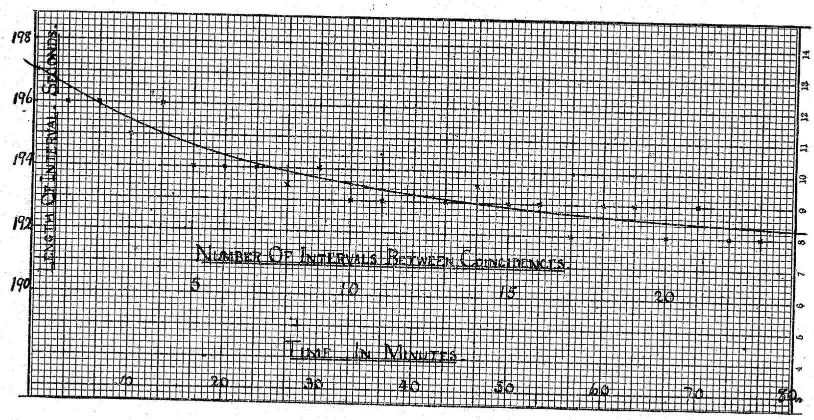

Figure 1.

This is a case in which a pendulum a little less than a meter long ran an hour and twenty minutes, starting with an amplitude of 46 mm. and finishing with an amplitude of 3.5 mm.

During this time the interval between coincidences decreased from about 197 seconds to about 192 seconds. Which value shall we accept? Neither of them, of course. The period is changing with the amplitude, and we really want the period for an infinitesimal amplitude. However, the interval between coincidences approaches a minimum as the curve approaches a position parallel to the X axis. The curve appears to be nearly horizontal at the end of the twenty-fourth interval, but just how much farther it has to run, it would be difficult to say.

Figure 2 shows the decrement in amplitude for the same set of observations. The two curves are quite similar in character.

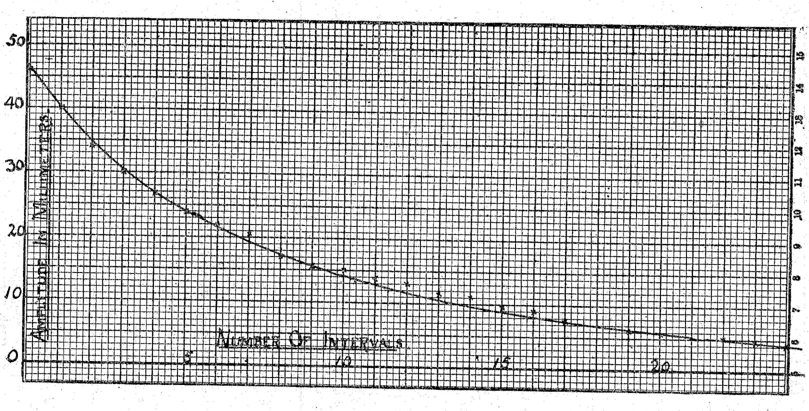

Figure 2.

In figure 3 they are plotted on the same scale, and when the two are placed side by side they are seen to be almost identical.

It is obvious that the interval approaches a minimum as the amplitude approaches zero. That is, when the curves are drawn on the same scale, the minimum on the interval curve may be taken at the X axis on the amplitude curve. In this case, the amplitude curve is two

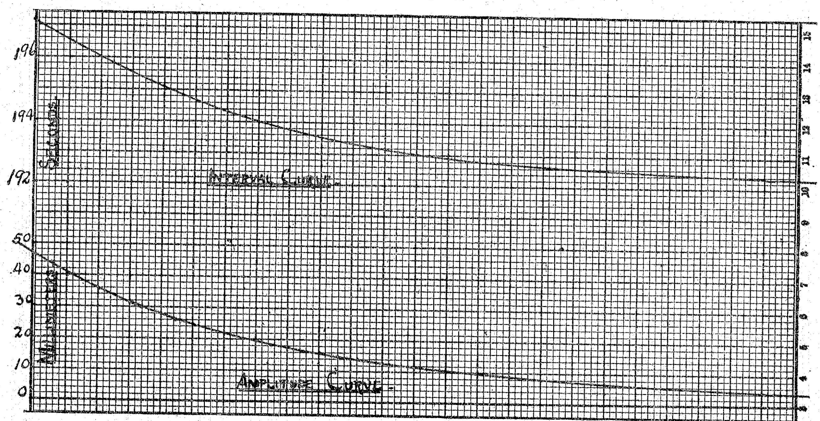

Figure 3.

spaces above the X axis at the end of the twenty-fourth interval. Since the two curves are so nearly identical, we may safely assume that the interval curve reaches a minimum two spaces below its position at the end of the twenty-fourth interval. This would be about 191.9 seconds, and with no very great uncertainty about the fourth figure.

Granting the possibility of an error of one or two units of the fourth order, let us see what the effect would be on the period.

For an interval of 191.9 seconds, the period is 191.9 divided by 192.9, or, .994816 if carried to the sixth figure. For an interval of 191.8 seconds, the period is 191.8 divided by 192.8, or, .994813+, and likewise for an interval of 191.7 seconds, the period is .994811—.

There is, of course, no justification for carrying these results to six figures; but the calculation shows that if the curve can be placed correctly to within one or two, or even to within three or four spaces on the chart, the period is correct to five figures.

These results are alike to within considerably less than one part in twenty thousand.

Our measurements, then, are sufficiently exact, and we might expect results within one part in ten thousand, if there are no other sources of error.

There is, of course, a formula which corrects the period of a pendulum for the amplitude, but who can say what other errors are to be corrected? For example, does the suspension bend exactly at the edge of the clamp which holds it, or does it begin to bend a little farther down? And, if the suspension is very slender, does the weight of the ball stretch the wire more when moving at a higher velocity than when moving at a lower velocity, and if so, how much does that add to the length of the pendulum?

These questions are important if the pendulum is swinging through an appreciable arc, but they lose their significance entirely when the pendulum is swinging through an infinitesimal arc, and therefore, errors arising from such sources are eliminated entirely by the method here described of finding the minimum interval between coincidences.

Finally, when all the errors are included in measuring both the length and the period of the pendulum, in the hands of our students in Knox College this method yields results ranging from 980.2 to 980.4.

The theoretical value of "g" for Galesburg is 980.26.