IMPROVED METHODS IN GRAVITY DETERMINATIONS

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Unquestionably, the most generally used method in the absolute determination of gravity is the pendulum method, in which the length, l, and the period, t, of a pendulum of one form or another, are the quantities to be measured.

To determine t with a degree of exactness which would give the value of "g" to the second decimal place, by counting the oscillations of a pendulum, would require continuous counting for considerably more than twenty-four hours. This, of course, would be a super-human task.

The well known coincidence method comes to our relief by enabling us to determine t over a long interval of time, from a few observations made at the beginning and end of as long a period of time as the pendulum will continue to oscillate. This may be as much as ten or twelve hours, for a well constructed pendulum swinging in a vacuum.

This method has been depended upon by practically all observers who have attempted important gravity measurements in the past. However, the results obtained are in error by an amount which is of serious importance when very exact work is attempted. The required value of t is the period of the pendulum at the end of its run, when it is swinging through an infinitesimal arc, and the coincidence method gives an average value for the entire time the pendulum is swinging. Of course, corrections can be made, but they are so numerous and some of them so uncertain, that there are, or at least ought to be, grave doubts concerning the final result.

In a paper presented at the Rockford meeting, and published in Vol. 15 of the Transactions of the Illinois Academy of Science, the writer described a method of determining the coincidence interval of a pendulum swinging through an infinitesmal arc, but we are now determining the period directly, by a more highly refined and much more satisfactory method.

In figure 1, the light from a 75 watt Mazda lamp, M, is concentrated by the lens, L, upon a narrow slit, S_1, in a metal plate behind the pendulum. A corresponding slit, S_2, in a thin metal plate attached to the pendulum itself, is carried back and forth in front of S_1 as the pendulum swings. When the pendulum is in the center of its path, a narrow beam of light flashes through the two slits and is picked up by a photoelectric cell, C_1. The current generated by this cell is amplified by a single stage resistance coupled amplifier, *A*, the details of which are shown in figure 2,

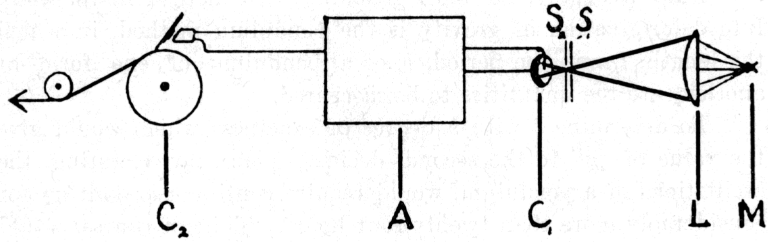

Fig. 1.

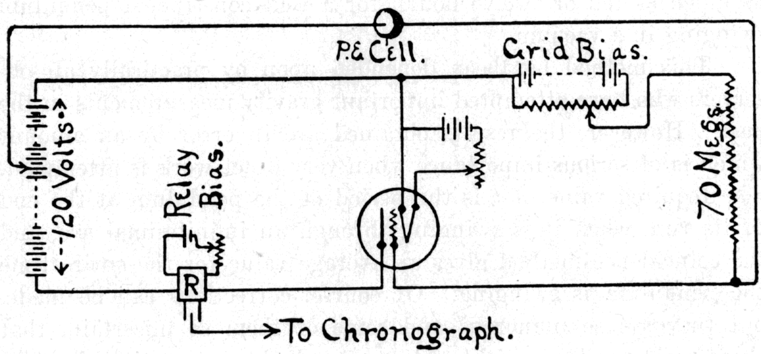

Fig. 2.

and the amplified current records the flash on the ribbon of a paper tape chronograph, C_2, in close proximity to a similar record in seconds, from a standard clock. As the length of a second on the ribbon depends upon the speed at which the chronograph is driven, a given length may be made to represent almost any fraction of a second. With a speed of about 50 cm per second it is quite possible to read the record to within .001 second, so that two

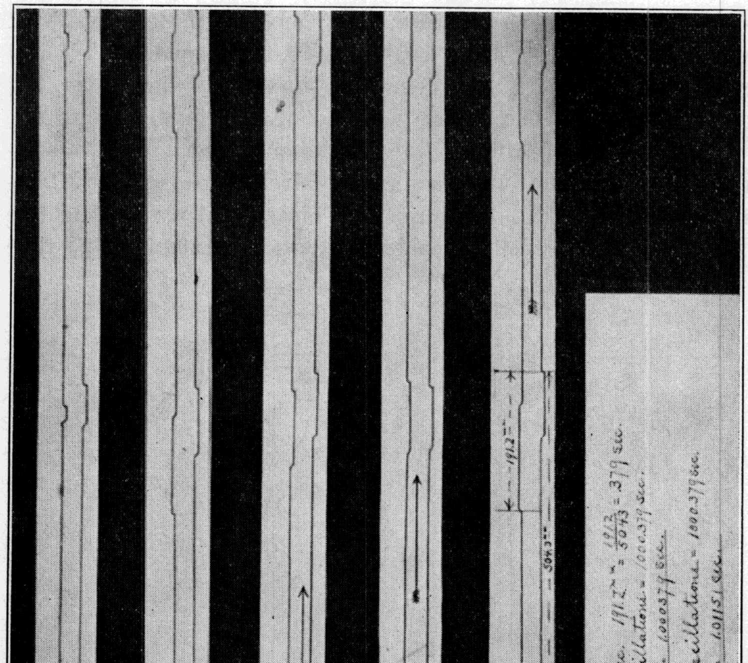

FIG. 3. Chronograph records: (1) large amplitude, (2) small amplitude, (3) a time rattle, (4) a coincidence, (5) a direct measurement of t.

records about 1,000 seconds apart enable us to determine the period of the pendulum to within one or two parts in a million, and the records can be obtained in 15 or 20 minutes instead of running the pendulum from 20,000 to 40,000 seconds. This last mentioned advantage is very great, not simply as a time saver, but because it enables us to determine the period when the amplitude is very small, and during an interval when the *change* of amplitude is imperceptible. With a very sensitive photo-electric cell furnished by the G. M. Laboratories of Chicago and a very sensitive relay, loaned by the Bell Telephone Co., we have found it possible

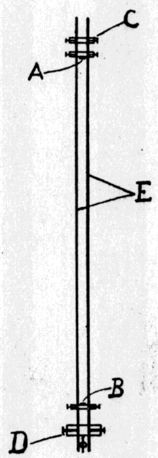

FIG. 4.

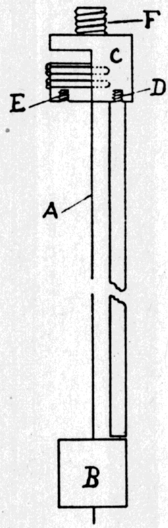

FIG. 5.

to pick up the signals when the amplitude of the pendulum is less than three-tenths of a millimeter. This, for a pendulum a meter long, is about one minute of arc and is well within the range in which *all* the corrections are negligible.

Figure 3 shows actual photographs of some of the chronograph records. Numbers 1 and 2 show the effect of varying amplitude. In No. 2 the amplitude is only .3 mm, but the "break" is as sharp as in No. 1, where the amplitude is 15 mm. No. 3 shows a "time rattle" interjected into the record by means of an independent key, for the purpose of indentifying some particular second. No. 4 shows a coincidence and No. 5 shows the method of determining the period by direct measurements of distances on the chronograph tape.

In determining the length of a pendulum, cathetometer measurements have been almost universally used in the past. Such measurements at best can not be better than about one part in 20,000. If, therefore, we are to determine "g" to within a few parts in a million, we must abandon cathetometer methods entirely.

In our present work we are comparing the length of a pendulum with that of a carefully prepared invar rod, nominally a meter long, the precise length of which is to be certified by the Bureau of Standards, or some other equally competent standardizing bureau.

We are using both simple pendulums and compound reversible pendulums.

Our method with a simple pendulum is as follows: Figure 4 represents a vertical section of a cylindrical pendulum bob, *B,* suspended by means of a bifilar suspension, *A,* from a rigidly supported clamp, *C.* The distance from the lower surface of the clamp to the upper surface of the bob is a trifle more than a meter. Our standard invar rod is provided with a screw at its upper end, and is securely screwed into the clamp at *D,* until the upper end of the rod is in contact with the lower surface of the clamp. The distance from the *lower* end of the rod to the upper surface of the bob is measured by means of a micrometer microscope. The bob is made in cylindrical form in order to facilitate these measurements. Optical tests show that the upper surface of the bob and the lower surface of the clamp are flat to within about half a wave length. The distance from the clamp to the bob is measured from *D* and also from *E,* and if there is any lack of parallelism between the surface of the clamp and the surface of the bob, the mean between the two measurements is taken as the distance at the center. To this is added half the length of the bob, a small negative correction for the effect of the mass of the fine wire suspension upon the center of gravity, and the correction for moment of inertia. The net result is the length, l, of the pendulum.

Figure 6 is a photograph of this pendulum mounted in its case, with the front cover removed and with the invar rod and the micrometer microscope in position for measuring the distance from the lower end of the rod to the top of the bob.

Figure 7 is a "close up" view of this last feature.

As the length of the invar measuring rod can be certified to within about a micron, and as the other measurements can easily

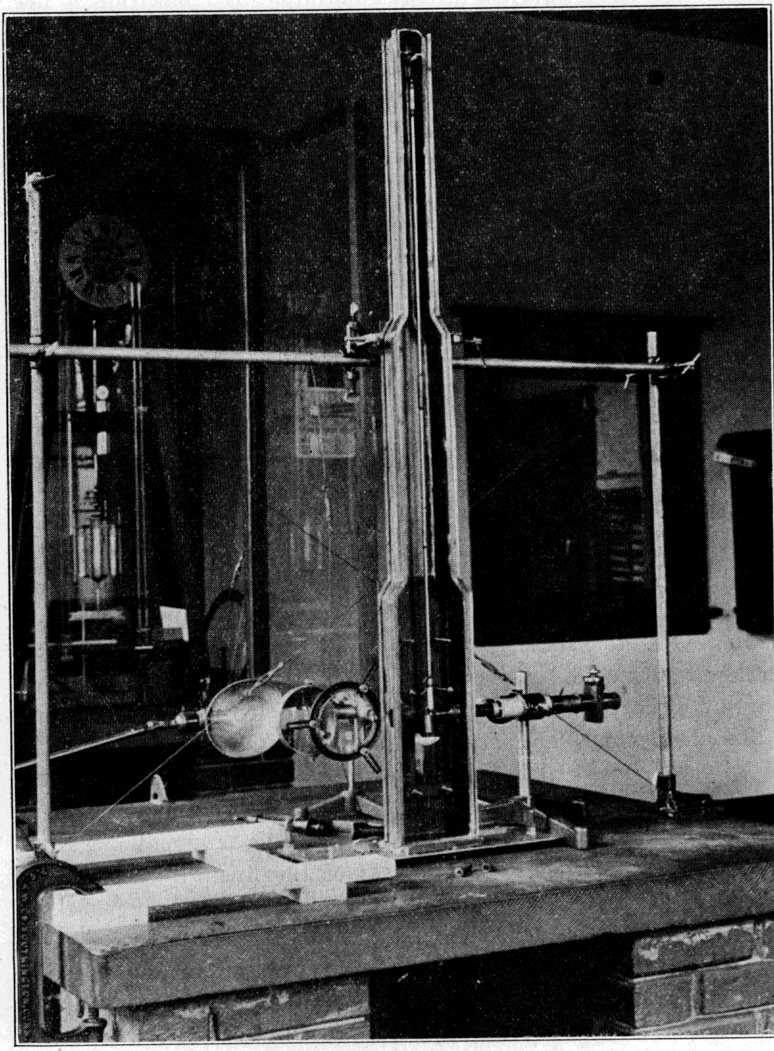

FIG. 6.

be made to within that degree of exactness, there should be no great difficulty in measuring l to within 2 or 3 parts in a million.

Our compound reversible pendulum has been designed with special reference to comparing its length with that of our invar measuring rod. Figure 5 shows the construction of this pendulum. *A* and *B* are the upper and lower knife-edge, and *C* and *D* are the upper and lower moveable weights. *EE* are the rods upon which the knife-edge blocks and the weights move. When these weights are so adjusted that the period of the pendulum is the same in the normal position as in the reversed position, then the length, l, is the distance between the knife-edges.

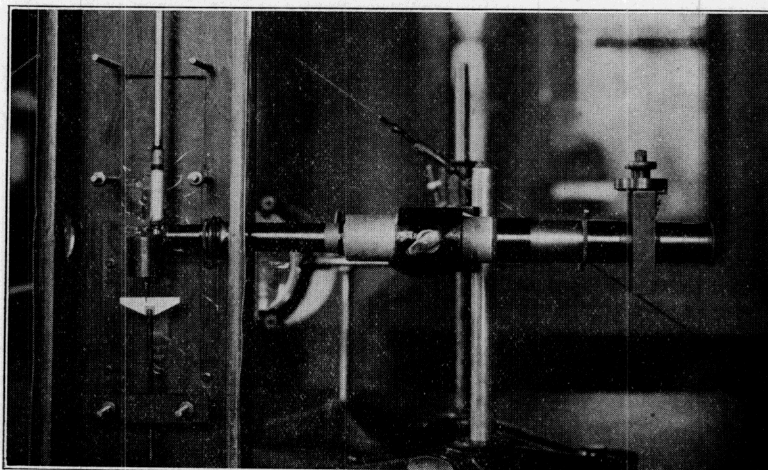

FIG. 7.

This distance is being measured in figure 8, where the pendulum is in a horizontal position and the invar measuring rod is lying between the knife-edges.

The upper end of the pendulum is held firmly in position by means of a screw passing through the upper knife-edge block. A cord is attached to the lower end, carried over a pulley and kept under tension by means of a weight attached to it. This weight is such that the length of the pendulum in a horizontal position is the same as its length in the vertical position, when swinging from its upper knife-edge.

To make the comparison, one end of the invar rod is brought into contact with the upper knife-edge and the micrometer microscope is set with its movable cross hair on the other end of the rod. The rod is then slipped lengthwise until it is in contact with the *lower* knife-edge and the movable cross hair is advanced to the new position of the end of the rod. This measures the amount to be added to the length of the rod to give the length, l, of the pendulum.

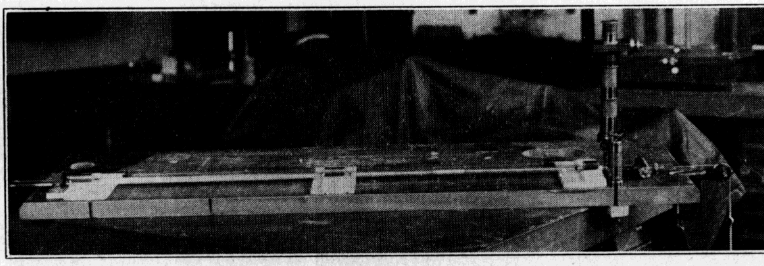

FIG. 8.

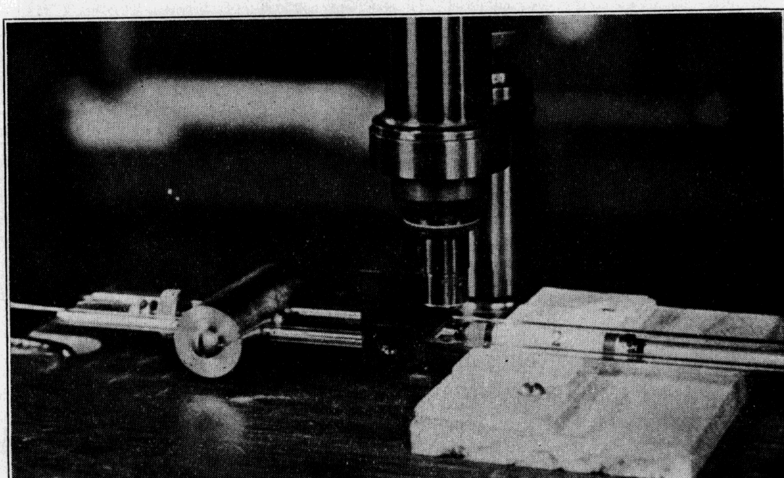

FIG. 9.

Figure 9 is a "close up" view of the end of the invar rod under the microscope.

Figure 10 shows the case in which the pendulum is mounted, and figure 11 is the same case with the front cover removed, showing the pendulum in position, swinging from its upper knife-edge.

As an important modification in the construction of a reversible pendulum, we are substituting roller bearings for knife-edges. The length, l, of a reversible pendulum, is the distance between the knife-edges. A simple enough proposition if the knife-edges are perfectly sharp, but unfortunately this condition can not be fully realized, and if realized, it could not be maintained. If a knife-edge were absolutely sharp, it would not remain so while in use.

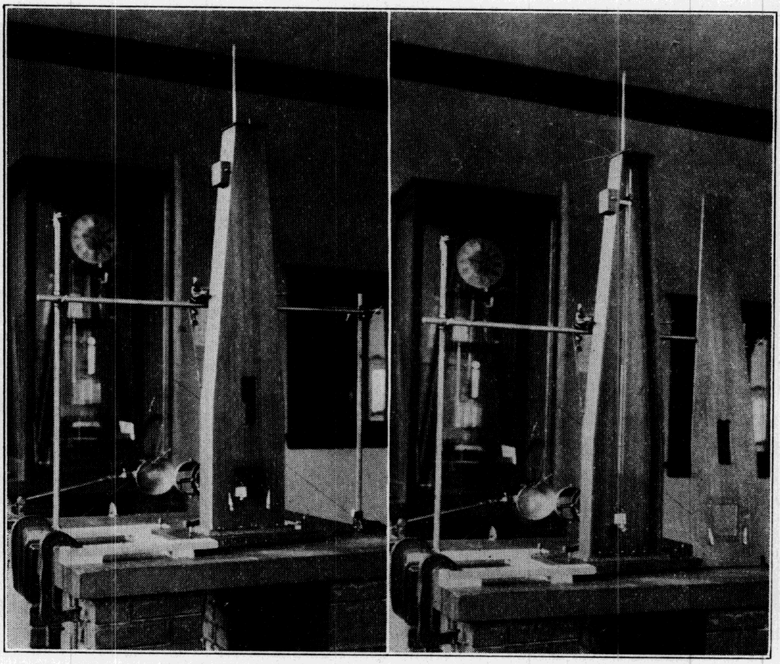

FIG. 10. FIG. 11.

If a good knife-edge be examined under a high power microscope it will be found as in figure 12, where we have at *A* represented by the dotted lines what is known as the "missing triangle." If this were all, the pendulum in swinging would simply bob up and down on the edges *e* and *f*. But as a knife-edge wears, it becomes more and more like a roller, and unfortunately its radius of curvature is not known. If the edge would wear into the form of a perfect cylindrical surface as at *C*, it would be more reliable.

If instead of attempting a perfect knife-edge, we substitute cylindrical bearings, or rollers, as at *D*, the length of our pendulum, which is now the distance between the centers of the rollers, is a perfectly definite and constant quantity, and the instrument has the advantage of being simple and easily constructed.

Knife-edges were assumed in the preceding description of our reversible pendulum, but we are actually using well-hardened, highly polished steel rollers, three or four millimeters in diameter. The distance between the surfaces of the rollers is measured with the invar measuring rod and micrometer microscope as described, and to this the diameter of one of the rollers is added. This finally gives the length, *l*, of the pendulum.

In order that the length of our pendulum shall remain constant, the rods *EE* in figure 7 should have an extremely low coefficient of linear expansion. For this reason, practically all modern investigators are using invar, a nickel-steel alloy which ex-

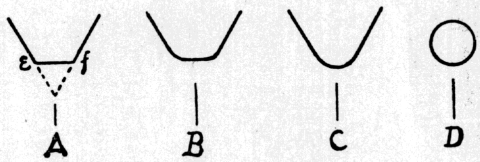

FIG. 12.

pands only about one part in a million, for a change of temperature of one degree. This is quite ideal, but invar is strongly magnetic, and the pull of the earth as a magnet is added to the earth's gravitational pull on the pendulum.

To avoid this disastrous complication we are recommending the use of fused quartz, which has even a lower coefficient of expansion than invar, and is entirely free from magnetic effects. In lieu of fused quartz, well-selected pyrex tubes give very satisfactory results.

It must be understood that the methods described in this paper relate to *absolute* determinations of gravity and not to the vast amount of *relative* work done in recent years with the half-seconds pendulum.

ACKNOWLEDGMENT

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