AN ANALYSIS OF RESPONSES MADE IN FOUR NARROW MENTAL FUNCTIONS INVOLVED IN THE COMPUTATION OF FRACTIONS

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This investigation is a study of the abilities or attainments of school children as measured by 35 pages of fractions covering every skill according to the extensive Knight-Ruch-Luse analysis of the same. This analysis includes 167 different skills in fractions and is an intensive and extensive sampling of the fraction field. Pupils in grades five, six, seven, and eight of the 46 grade schools of the city of DesMoines were used in this experiment. The 35 pages of fractions contained 590 problems from the simplest to the most complex, and the total number of pupil attempts was 264,000.

The aims of this study in its entirety were: first, to ascertain how accurately the first teaching of fractions has been done in all the fraction skills and how much of an increase is made in acquiring these skills as the children progress through the grades; second, to determine what types of errors children make in the four processes in fractions; and, third, to explain why these different types of errors are made in the light of what is taught in the different textbooks analyzed. The writer will devote most of this paper to the second problem, despite the fact that nine textbooks were carefully analyzed with regard to the number of problems in each process and skill and the degrees of efficiency in the different processes were extensively analyzed for the different grades.

Before the study of type errors was attempted, a very careful study was made to ascertain if possible every kind of error that children make in doing fraction problems. The writer and his assistants, after charting errors for several months, became familiar with all the errors made by the children investigated in this field. Charts were made showing every possible error that a child could make and the number of the page where each error was made. From comparisons of these the type errors were then charted. It was found that from 16 to 22 errors were made in each of the four fraction processes. Before a process was attacked the writer and his assistants carefully discussed the type errors in that process to eliminate the personal element as much as possible. Table I is

a summary of the type errors of greatest frequency grade by grade and the average percentage for each.

TABLE I

PERCENTAGES OF TYPE ERRORS OF GREATEST FREQUENCY IN THE COMPUTATION OF FRACTIONS BY SCHOOL CHILDREN

ADDITION

Grade—	5	6	7	8	Average
Common denominator	36.2	26.1	24.8	24.8	28.1
Reduction	23.0	28.4	29.1	21.2	25.0
Addition	24.4	31.4	31.3	35.5	31.0
Total percentage	83.9	85.9	85.2	81.5	84.1

SUBTRACTION

Grade—	5	6	7	8	Average
Borrowing	24.0	23.4	28.0	25.2	24.9
Wrong process	20.5	15.3	12.4	8.6	15.1
Subtraction	19.9	19.2	17.6	20.5	19.2
Reduction	10.3	17.6	18.7	21.0	16.2
Common denominator	12.6	13.5	11.7	13.7	12.8
Total percentage	87.3	89.0	88.4	89.0	88.2

DIVISION

Grade—	5	6	7	8	Average
Inversion	25.9	28.5	32.3	39.6	31.6
Reduction	40.3	42.1	43.5	35.3	40.4
Multiplication	9.7	8.4	8.0	6.5	8.1
Addition	4.9	3.5	2.0	3.0	3.2
Cancellation	5.9	8.6	7.4	7.1	7.4
Total percentage	86.7	91.1	93.2	91.6	90.7

MULTIPLICATION

Grade—	5	6	7	8	Average
Reduction	55.3	49.2	56.0	47.6	52.1
Wrong process	11.7	20.3	13.2	19.8	16.3
Multiplication	17.2	13.8	16.7	16.2	15.8
Cancellation	3.6	5.0	4.5	5.5	4.6
Total percentage	87.8	88.5	90.4	89.1	88.8

The rank correlations, for each process on the type errors, grade against grade, are given in Table II. The correlation is much lower than it should be in addition and subtraction because of the nature of the problems and because the cards 25 to 35 are short in the fifth grade. The type errors made in the fifth grade are correlated against the total type errors of the sixth, seventh, and eighth grades; and the total type errors of the sixth, seventh, and eighth grades against totals of the fifth, sixth, seventh, and eighth grades because of this shortage in the fifth grade. The correlations are shown in Table II.

TABLE II

CORRELATIONS

Grades	Subtr.	Add.	Mult.	Div.
5–6	.707	.803	.823	.917
5–7	.756	.697	.842	.889
5–8	.727	.727	.871	.871
6–7	.908	.775	.908	.973
6–8	.900	.727	.945	.936
7–8	.927	.880	.936	.945
5–6, 7, 8	.775	.756	.905	.917
6, 7, 8–5, 6, 7, 8	.973	.954	.982	.991

The rank correlations for eight of the fraction cards containing problems in the four processes were worked out on the basis of the examples right, grade by grade, and are given in Table III.

TABLE III

CORRELATIONS

	Card	5–6	5–7	5–8	6–7	6–8	7–8
Addition	8	.945	.860	.936	.870	.917	.973
Subtraction	12	.954	.908	.927	.908	.889	.840
Multiplication	20	.973	.973	.936	.982	.945	.945
Mixed	21	.954	.954	.936	.991	.917	.936
Division	24	.954	.908	.954	.945	.945	.926
Mixed	25	.954	.889	.823	.945	.900	.964
Addition	27	.852	.927	.889	.908	.964	.954
Subtraction	31	.918	.852	.789	.964	.880	.936

The writer does not maintain that there would be such a high correlation on the cards containing problems of equal difficulty. In fact the cards which contained fractions with the greatest number of fraction skills and the widest range of difficulty were deliberately chosen to show that there was a relatively high correlation between the difficulty of the different problems grade by grade or, in other words, that the hardest problems in one grade were, generally speaking, the hardest problems in every other grade.

SUMMARY AND RECOMMENDATION*

1. The commonest errors are made in the fundamental fraction skills. This must be due to the fact that the amount of practice given in the textbooks is not sufficient to over-learn these necessary bonds.

2. These basic fraction skills, such as reduction, the common denominator, borrowing, and so forth, are evidently not emphasized sufficiently in the textbooks; furthermore there is little provision made for any sort of maintenance program.

* This summary covers the entire study and not merely the points suggested in this paper.

3. There is a wide variation of practice in all the textbooks analyzed concerning the amount of time to be spent on the different processes and the different fraction skills.

4. There seems to be little consistency of effort to the formation and maintenance of the bonds which will make for proficiency in fractions.

5. The improvement that children make through the grades seems to be more the result of good teaching and fraction drives than of the material provided in the textbooks.

6. Fractions could be made much easier for the average child if we would teach the specific bonds which the child should use and no others, and insist that the errors that children make should be corrected. (Each individual should give a great deal of time after the first teaching to practice in his own individual weaknesses.)

7. The fraction examples tend to hold the same general position of relative difficulty throughout the four upper grades.

8. The same types of errors tend to appear with the same relative frequency throughout these grades.

9. Within the limits of this study, subtraction of fraction is the hardest process and is followed in order by multiplication and division, with addition perceptibly the easiest.

10. Three to five kinds of types of errors or omissions account for 84-90% of the errors children make in fractions in each process.

11. Proficiency in fraction depends on: (a) making the first teaching adequate; (b) providing a proper maintenance program; (c) keeping careful records of the individual performance of each pupil; (d) careful practice by each pupil to overcome his own individual difficulties; (e) the right kind of diagnostic material and the kind of remedial work to find out accurately what these individual difficulties are and to correct them; and (f) frequent opportunities for diagnostic testing of these individual weaknesses.

SUGGESTED PROBLEMS FOR FUTURE STUDY

1. Failure to do cancellation as a function of the difficulty in multiplication and division.
2. Failure to reduce to L. C. D. as a function of difficulty in addition and subtraction.
3. Interference factors in the fraction processes and the best ways of removing them.
4. Experiments on what to do regarding (or to remedy) the major types of errors in fractions.

5. Experiment on comparative difficulties of different denominators or different combinations used in fraction processes, especially addition and subtraction.
6. Extensive experiment on the nature of the whole-number difficulties in their relation to the fraction errors.
7. Influence of the errors children make in whole numbers, by a pre-test given in these whole-number processes, upon errors made in various fraction processes. (An especially valuable Ph. D. thesis.)
8. Fractions necessary to use in life, as obtained from a sampling of fractions used in each type of business.
9. How best to make the study of fractions function in applying results obtained in Problem 8.
10. Methods of obtaining a common basic fraction vocabulary for our course of study by analyzing the fractions used in textbooks and the methods of teaching the same, then reconciling or applying them to life problems in which fractions are used.
11. Analysis of mistakes that occur in fractions by taking each type of error that occurs in this study and putting it in an example worked out to see what percentage of time children take to recognize this type of error. An attempt might be made to make a portion of this study by the case-book method.
12. A problem which seems very important could be worked out with material used in this study. Errors made by each percentile. This would emphasize the necessity of remedial work for the lower tenth percentile, as most of the errors and no attempts would be made by them. Perhaps the most important thing would be the portion of the work not attempted by the lower tenth percentile and the relation of correctness to amount done.
13. The correlation of errors children make in various fraction skills and the number of times such fraction skills are practiced in textbooks used. A controlled experiment.