

A SUMMARY OF EXPERIMENTAL INVESTIGATIONS  
IN ARITHMETIC

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The summary of investigations in arithmetic, of which this paper is an abstract,<sup>1</sup> is an attempt at synthesis of the findings of the experimental research relating to instructional methods in this subject, with recognition of their varying degrees of dependability. The criteria used in the evaluation of the experiments are as follows: (1) definition and restriction of the experimental factor; (2) control of pupil factors (equivalence of groups); (3) control of important non-experimental factors; (4) accuracy and validity of measures of differences in achievement; (5) justification of generalization. The extent to which the techniques used and the data collected in each of the experiments satisfied these criteria were taken as an indication of the dependability of the conclusions.

The summary is divided into the six major divisions as follows: (1) methods of learning and teaching the fundamentals; (2) methods of drill in the fundamentals; (3) methods of teaching verbal problems; (4) methods of providing diagnosis and remedial treatment; (5) methods of teaching the reading of arithmetic subject-matter; (6) methods of motivating learning activity in arithmetic. Space is too limited in the present paper to consider, in detail, the experimental research under each of these heads. The conclusions of the more interesting studies have been selected for presentation along with some general criticisms which may be made of the experimental research in this field.

*Methods of learning and teaching the fundamentals.* Buckingham has reported that it is more effective to teach pupils to add downward rather than in the upward direction. Recent research on transfer of training in arithmetic, notably that of Beito and Brueckner, appears to indicate that it is immaterial which direction is specified in teaching pupils to add. The conclusions of several investigations in the field of subtraction appear to favor the subtractive method of subtraction

<sup>1</sup> The summary is to be published as "A critical summary of research relating to the teaching of Arithmetic," Bull. 58, of the Bureau of Educational Research of the University of Illinois with Walter S. Monroe and Max D. Engelhart as joint authors.

rather than the additive method, and that the method of equal addition or carrying, in connection with the subtractive procedure, is more effective than decomposition or borrowing. None of the various methods of subtracting appear, however, to be significantly different in effectiveness. The conclusions of the experiments on methods of learning and teaching division, fractions, decimals, percentage, proportion, and denominate numbers are of so little dependability as to be unworthy of mention here.

*Methods of drill in the fundamentals.* Systematic drill in the fundamentals has been shown by several experiments to be highly effective. It has been shown to be more effective than incidental instruction such as occurs when the project method is used. The conclusions of G. M. Wilson indicate that a combination of systematic drill, and the less formal incidental instruction, is probably most effective as the incidental instruction contributes motivating influences while the systematic drill provides the distribution of practice essential if recognized objectives of arithmetic are to be attained.

Attention should be called in this connection to the significant contributions of research on the specificity of the calculation abilities, and on the relative difficulties of learning the different number combinations. These contributions have made possible the construction of drill materials shown experimentally to be more effective than those prepared informally by teachers. It seems reasonably true that drill materials which provide practice on the number combinations, proportionate to their difficulty, as revealed by the research just referred to, is more effective than drill material not prepared in this way. It also seems reasonably true, from the reported conclusions of a number of experiments, that drill materials should distribute practice over the number combinations, should provide practice on the higher decades in addition, and that it is relatively unessential to provide practice on number combinations in the reverse order. For example, if pupils are drilled on  $6 + 3 = 9$  they will learn without other provision, according to Olander, that  $3 + 6 = 9$ . It is also reported that drill materials of a mixed nature are an effective means of securing maintenance of skill in the fundamentals.

*Methods of teaching verbal problems.* Monroe has concluded in a recent investigation that pupil responses to verbal problems are usually characterized by failure to employ critical reflective thinking. The findings of this, and other investigations, tend to show that pupils are most likely to make correct responses where problems are stated in familiar rather than unfamiliar terminology, when the problem statement is concrete rather than abstract or imaginative, and when irrele-

vant elements are not included. Systematic training in finding the data given in the problem, in deciding upon the calculations to be made, and in estimating the answer in round numbers are reported to be effective procedures for teachings pupils to solve verbal problems.

*Methods of providing diagnosis and remedial treatment.* Comprehensive and reasonably dependable experimental evidence supports the conclusion that diagnostic and remedial treatment is highly effective in the field of arithmetic. Conclusions have been reported which favor both individual diagnosis in which "first-hand observation is made of the actual work of the pupil" and diagnosis by means of diagnostic tests. Further research is needed before it may be concluded which method is relatively more effective. Expertly prepared drill material, providing practice for abilities diagnosed as weak, has been shown to be an effective means of providing remedial treatment.

*Methods of teaching the reading of arithmetical subject matter.* Reading ability has been shown to be an important factor in arithmetic achievement. The eye-movement studies of Buswell and John and of Terry should be referred to in this connection. While it appears certain that reading ability is an important factor in arithmetic achievement, further research is needed before a statement may be made with respect to the nature of its influence in the different situations represented by calculation examples and verbal problem statements of different types.

Dramatization and story-telling in connection with instruction in verbal problems; general training in reading, and the use of solution sheets containing information with respect to how problems should be read, and spaces for recording of data useful at different stages in the solution of the problem have been shown to be feasible means of teaching pupils to read verbal problems correctly. The research supporting this statement is very crude in nature. Further research must be conducted before dependable suggestions may be made with respect to the most effective methods of teaching pupils to read arithmetical subject matter.

*Methods of motivating learning activity in arithmetic.* Informing pupils of definite goals of achievement to be attained, and of their progress toward these goals, has been shown to be an effective means of stimulating learning activity in arithmetic. Competition, particularly individual rather than group competition, has been reported effective in securing intense effort in arithmetical learning activity. The use of this device appears justified in attempting to bring a class out of a slump in learning. Prolonged, or repeated, use is probably unjustified as the pupils are likely to acquire the attitude that victory in competi-

tion is the goal rather than the acquisition of arithmetical ability. Commendation and reproof have both been shown to be effective stimulants to learning activity. In arithmetic commendation appears to be more effective for most pupils. Reproof appears justified, however, where pupils are careless in the use of skills which they are known to have acquired. In closing, it should be stated that the most effective means of motivating learning activity in arithmetic is to request appropriate learning activity. Learning exercises should challenge the abilities of the pupils.

*A general criticism of the experimental research on methods of teaching arithmetic.* A critical examination of the experiments reported in this field indicates that few approach acceptability when measured by our criteria. In most cases, the experimental factors were complex in character and were not specified with precision. The more recent experiments seem satisfying with respect to equivalence of groups, but the large majority of the experiments may be criticized in this respect. Failure to control important non-experimental factors, particularly the zeal and skill of the teacher, is a criticism which applies to most of the experiments. Lack of validity is characteristic of many of the measurements made of achievement. Frequently, this lack of validity tends to make differences in achievement erroneously in favor of a given method. Finally, the characteristic use of small groups of pupils makes generalization in most instances unjustifiable. It may be concluded, therefore, that the findings of research in this field must be regarded as largely suggestive in nature, and much precise experimentation remains to be done before rules, principles, or laws respecting the teaching of arithmetic can be established.