WAVE FORMS OF PHASE-SHIFTED SINE PULSES AND THEIR APPLICATIONS

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In the course of development of an automatic recorder of spectral sensitivities of photoelectric cells, means were required for producing pairs of 60 cycle modulated light sources whose intensities could be sustained in definite phase relations. This led to a theoretical study of rectified phase-shifted sine waves. Enhanced by an urgent demand for a practical method of precise determination of small phase shifts this work has been further developed experimentally. The main theoretical and experimental results are given in the following brief report.

The task set for the theoretical investigation consisted of finding new wave forms characterized by sharply marked critical points whose coordinates would determine the phase shifts. Such new wave forms were derived by the superposition of two rectified phase-shifted sine waves.

Three distinct wave forms have thus been studied:

(1) Those obtained by the subtraction of two rectified phase-shifted sine waves.

(2) Those obtained by the addition of two rectified phase-shifted sine waves.

(3) Those obtained by a rectified phase-shifted sine wave added to or subtracted from a full sine wave.

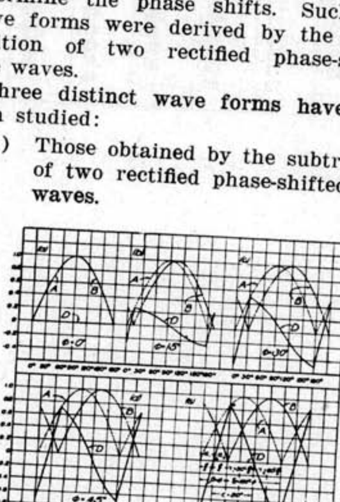

Fig. 1.—Graphical subtraction of two rectified phase-shifted sine waves.

Due to space limitation only the first type of wave form will be discussed here.

Fig. 1 represents graphically the subtraction of instantaneous values of two phase-shifted rectified waves, A and B, of equal amplitude and frequency. Each of the seven graphs shows the resultant wave shape D=A–B for a particular phase shift ϕ, namely, $\phi = 0°$, 15°, 30°, 60°, 90°, and 120° respectively. The resultant curve D appears as an alternating wave of double the frequency of the original sine wave. The curve has sharp peaks marked a, c, and e. It crosses the X-axis at the points b and d. Interestin from the point of view of practical application is the property which makes the amplitude of the curve D proportiona to the sine of the phase angle ϕ. A further property is that for D=O, $\alpha_{b,d} = K\pi/2 + \phi/2$, where K=0, 1, 2, . . . anc that the value of the abscissa for th maximum is $X_c = K\pi + \phi$ and for th minimum, $X_{a,e} = K\pi$. With increasin phase shifts the wave form develop gradually from an unsymmetrical shap with a small amplitude into a symmetrical triangular form of maximal amplitud at $\phi = 90°$. Similar studies of the tw other types of wave forms mentione under (2) and (3) reveal other characteristics also depending on the phas angle.

In order to examine the possibility o producing such wave forms in electrica circuits, the oscillographic method wa applied in connection with mixing ci cuits for combining the two rectifie phase-shifted sine waves.

Fig. 2 is an example of a series of osci lograms thus obtained for $\phi = 0°$, 15 30°, 45°, 60°, and 90° respectively. Th left column shows the component rectifie

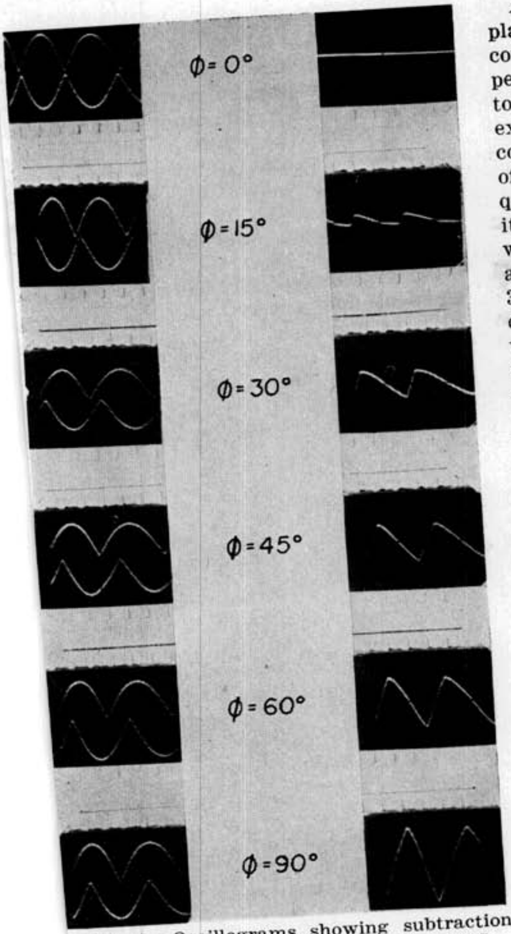

Fig. 2.—Oscillograms showing subtraction of two rectified phase-shifted sine waves.

sine waves each taken by a separate exposure while the second column of oscilograms shows the resultant forms changing as the phase is varied step by step. These correspond closely to the curves theoretically predicted.

In order to produce any required defnite combination of currents it was necessary to develop accurate means for adjusting phase angles of the component currents. The oscillograms shown in Fig. 2 were obtained at 60 cycles per econd by a known method of addition of electrical vectors in quadrature relaion.

For audible frequencies from f = 500 ycles to f = 10,000 cycles per second a more accurate method of obtaining phasehifted currents was developed.

As shown in Fig. 3, in room I were placed the oscillator O, whose frequency could be varied from 20 to 20,000 cycles per second. The oscillator was coupled to a transmitter T. For frequencies not exceeding 2,000 cycles this transmitter T, consisted of a high impedence earphone of the Baldwin type. For higher frequencies it was replaced by a high fidelity driving unit of a dynamic speaker whose voice coil impedance was 16 ohms and whose audio spectrum extended from 30 to 10,000 cycles. At a distance d_1 and d_2 of approximately two wave lengths were placed two high impedance headphones P_1 and P_2, identical in their characteristics. One of the headphones, P_2, was fixed in position. The other, P_1, was mounted on a stand and could be shifted between guides by means of a worm-gear G. By turning a crank K, the distance d_1 could be varied and set to any desired position with an accuracy of ± 0.2 mm. For each given phase angle ϕ, the distance $d = d_1 - d_2 = \phi v/\pi f$ could be calculated from the sound velocity v and frequency f and then adjusted accordingly.

In room II were placed two sets of measuring equipment. Each set consisted of a voltage amplifier (V_a or V_b), a matching transformer (W_a or W_b), a full-wave rectifier (Y_a or Y_b) and a coupling circuit (Z_a or Z_b).

The outputs, a and b, of the receiver headphones P_1 and P_2 were fed through shielded concentric cables into the re-

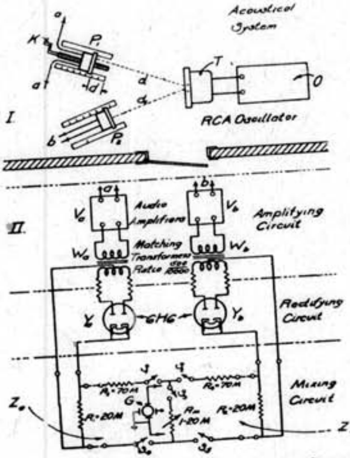

Fig. 3.—The acoustic phase-shifting apparatus.

spective amplifiers V_a and V_b. An oscillograph G was used as an indicating instrument to observe the resultant wave forms.

For frequencies from 10,000 to 100,000 cycles the method of adjusting circuit constants was used for producing phase shifts. Thus, the entire frequency spectrum over the range from 30 to 100,000 cycles was found adaptable in connection with the described wave forms.

The applications of the new wave forms are manifold. By direct use of an oscilloscope which records the wave forms on a screen, it is possible to measure the coordinates of the critical points of the resultant wave forms and thereby determine phase differences. However, more refined methods of utilizing the characteristics of the new forms are being developed. Of other applications which are in the course of development, the following may be mentioned as examples: time axis sweep circuits for oscillograph and television tubes, square wave generators, and harmonic oscillators.

A complete report concerning the mathematical and experimental investigation will be published in a Bulletin of the University of Illinois' Engineering Experiment Station.